1.

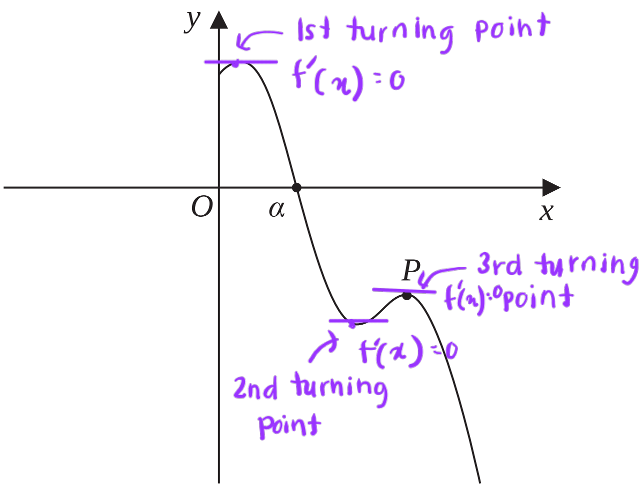

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 8\sin\left(\frac{1}{2}x\right) - 3x + 9 \qquad x > 0$$

and x is measured in radians.

The point P, shown in Figure 2, is a local maximum point on the curve.

Using calculus and the sketch in Figure 2,

(a) find the x coordinate of P, giving your answer to 3 significant figures.

(4)

The curve crosses the x-axis at $x = \alpha$, as shown in Figure 2.

Given that, to 3 decimal places, $\mathrm{f}(4) = 4.274$ and $\mathrm{f}(5) = -1.212$

(b) explain why α must lie in the interval $[4, 5]$

(1)

(c) Taking $x_0 = 5$ as a first approximation to α, apply the Newton-Raphson method once to $\mathrm{f}(x)$ to obtain a second approximation to α.

Show your method and give your answer to 3 significant figures.

(2)

a) $f(x) = 8\sin\left(\frac{1}{2}x\right) - 3x + 9$

$f'(x) = 8 \times \frac{1}{2}\cos\left(\frac{1}{2}x\right) - 3$ 1

$= 4\cos\left(\frac{1}{2}x\right) - 3$ 1

at P, $f'(x) = 0$

$4\cos\left(\frac{1}{2}x\right) = 3$ (1)

$\cos\left(\frac{1}{2}x\right) = \frac{3}{4}$

$\frac{1}{2}x = \cos^{-1}\left(\frac{3}{4}\right)$

$2\pi - 0.7227$ $2\pi + 0.7227$

$\frac{1}{2}x = 0.7227, 5.560, 7.006$

$x = 1.445\ldots, 11.120\ldots, 14.011\ldots$

$\therefore P_x = 14.0$ (3 s.f.) (1) 3rd turning point

b) $f(4) = 4.274 > 0$ and $f(5) = -1.212 < 0$ (1)

There is a change of sign between $f(4)$ and $f(5)$.

The function is continuous in interval $[4,5]$

$\therefore 4 < \alpha < 5$

c) Newton Raphson

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$x_0 = 5$

$$x_1 = 5 - \frac{f(5)}{f'(5)}$$

$$= 5 - \frac{8\sin\left(\frac{1}{2}\times 5\right) - 3\times 5 + 9}{4\cos\left(\frac{1}{2}\times 5\right) - 3} \quad (1)$$

$x_1 = 4.80462$

$= 4.80 \ (3 \text{ s.f.}) \quad (1)$